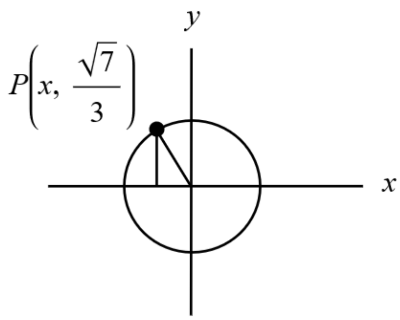
Exercise 137

For the following exercises, P is a point on the unit circle. a. Find the (exact) missing coordinate value of each point and b. find the values of the six trigonometric functions for the angle θ with a terminal side that passes through point P. Rationalize denominators.

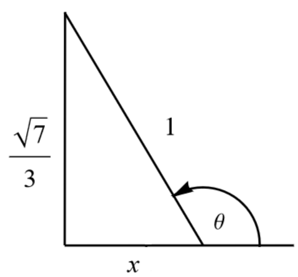
$$P\left(x, \frac{\sqrt{7}}{3}\right), \ x < 0$$

Solution

The given point P on the unit circle is shown below. $x < 0$ means that it's in the left half.



Zoom in on the right triangle formed by P. θ is the counterclockwise angle from the positive x-axis.



The hypotenuse has a length of 1 because P is on the unit circle. The sides of a right triangle are related by the Pythagorean theorem, and this allows us to determine x.

$$x^2 + \left(\frac{\sqrt{7}}{3}\right)^2 = 1^2$$

$$x^2 = 1^2 - \left(\frac{\sqrt{7}}{3}\right)^2$$

$$x^2 = \frac{2}{9}$$

$$x = -\frac{\sqrt{2}}{3}$$

Therefore, the six trigonometric functions are

$$\sin\theta = \frac{\frac{\sqrt{7}}{3}}{1} = \frac{\sqrt{7}}{3}$$

$$\cos\theta = \frac{x}{1} = x = -\frac{\sqrt{2}}{3}$$

$$\tan\theta = \frac{\frac{\sqrt{7}}{3}}{x} = \frac{\frac{\sqrt{7}}{3}}{-\frac{\sqrt{2}}{3}} = -\sqrt{\frac{7}{2}} = -\frac{\sqrt{14}}{2}$$

$$\csc\theta = \frac{1}{\frac{\sqrt{7}}{3}} = \frac{3}{\sqrt{7}} = \frac{3\sqrt{7}}{7}$$

$$\sec\theta = \frac{x}{1} = x = -\frac{3}{\sqrt{2}} = -\frac{3\sqrt{2}}{2}$$

$$\cot\theta = \frac{x}{\frac{\sqrt{7}}{3}} = \frac{-\frac{\sqrt{2}}{3}}{\frac{\sqrt{7}}{3}} = -\sqrt{\frac{2}{7}} = -\frac{\sqrt{14}}{7}.$$